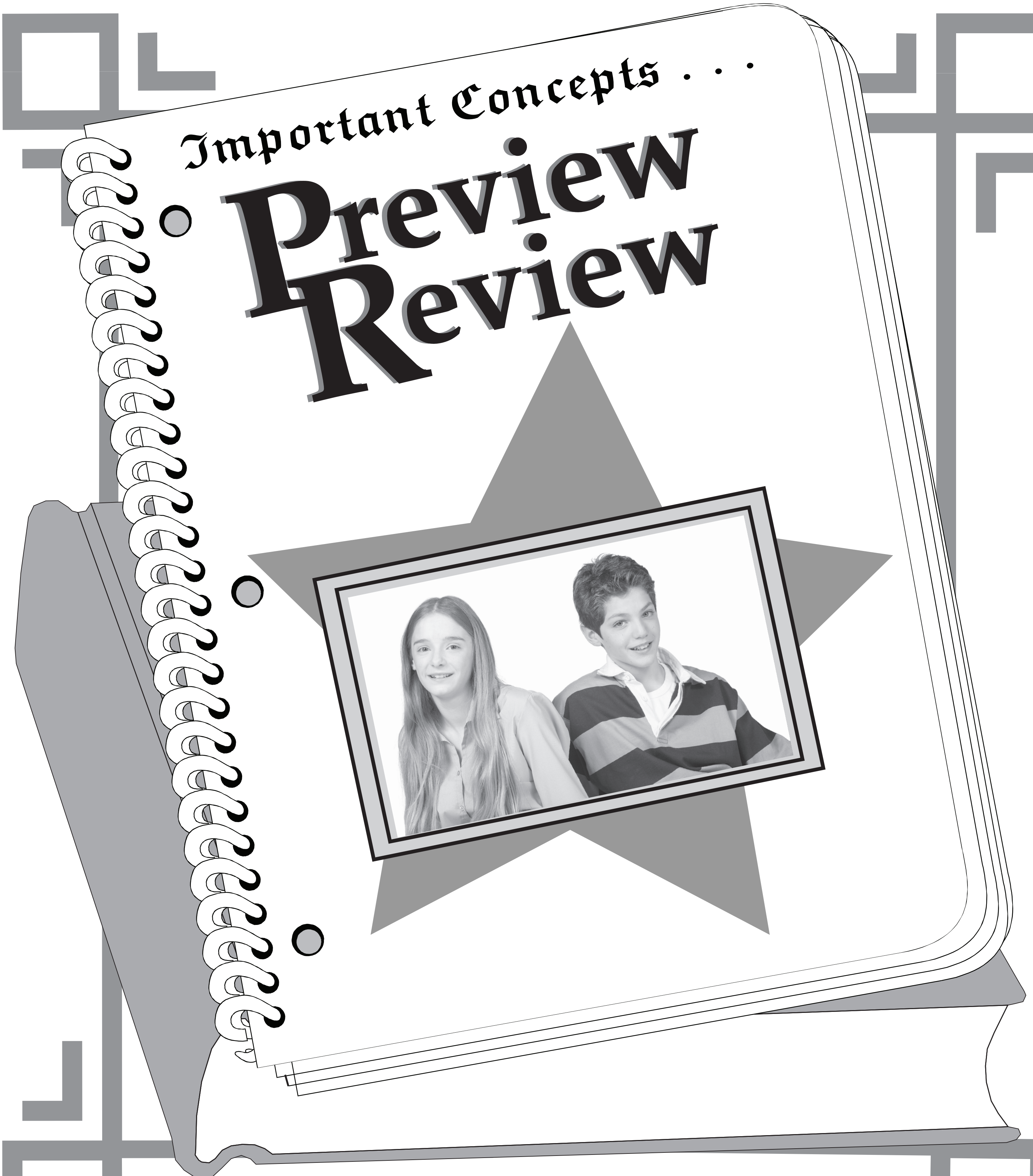

Mathematics Grade 8 TEACHER KEY

W1 - Lesson 2: Working with Ratios and Rates

Important Concepts of Grade 8 Mathematics

Materials Required

Protractor
Ruler
Calculator

No Textbook Required

This is a stand-alone course.

Mathematics Grade 8
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Preview/Review Concepts for Grade Eight Mathematics

Teacher Key

W1 – Lesson 2:

Working with Ratios and Rates

OBJECTIVES

By the end of this lesson, you will be able to:

- Express a two-term ratio in different forms
- Express a three-term ratio in different forms
- Express a part-to-part ratio as a part-to-whole ratio
- Express a ratio as a percent
- Solve problems involving ratios
- Solve problems involving rates and unit rates

GLOSSARY

Ratio – a comparison of two or more values using the same units.

Part-to-part ratio – a ratio that compares one part of a collection to another part of a collection.

Part-to-whole ratio – a ratio that compares a part of a collection to the entire collection.

Rate – a comparison of amounts or measurements using different units.

Unit Rate – a rate with the second term being 1.

W1 – Lesson 2: Perfect Squares and Square Roots

Materials required:

- Paper, Pencil, and Calculator

Part 1: Ratios and Equivalent Ratios

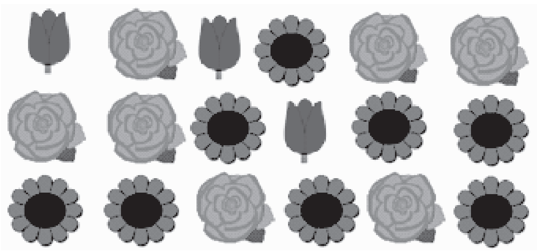
A ratio is a comparison of two or more values that both use the same units. For example, a ratio can be used to compare different types of flowers in the garden or different colours of jellybeans in a candy jar.

Ratios can be expressed as two term ratios, such as 2 : 5 or 2 to 5. They can also be expressed as three term ratios, such as 1 : 3 : 7 or 1 to 3 to 7.

Example 1

Marshall plants the following flowers in his garden.

a. Write the ratio of roses to tulips



number of roses : number of tulips

The ratio can be expressed as 7 : 3 or as 7 to 3.

Since the ratio compares the number of roses to the number of tulips, the first term in the ratio must represent the number of roses and the second term must represent the number of tulips. A ratio that compares the number of roses to the number of tulips will be different from a ratio that compares the number of tulips to the number of roses.

The ratio of tulips to roses would be 3 : 7 or 3 to 7.

This is a part-to-part ratio as it compares only roses to only tulips.

b. Write the ratio of tulips to all of the flowers in the garden.

number of tulips : total number of flowers

$$3 : 18$$

$$\frac{3}{3} : \frac{18}{3}$$

$$1 : 6$$

Simplify each ratio.

The ratio can be expressed as 1 : 6 or 1 to 6. Always express ratios in lowest terms.
This ratio is called a part-to-whole ratio as it compares only tulips to all the flowers in the garden.

c. Write a ratio that compares the number of tulips to daisies to roses.

number of tulips : number of daisies : number of roses

$$3 \quad : \quad 8 \quad : \quad 7$$

The ratio can be expressed as 3 : 8 : 7 or 3 to 8 to 7. Since none of the three terms share a common factor, this ratio is already in lowest terms.

d. Express the number of daisies to the total number of flowers in the garden as a percent.

A ratio can be expressed as a percent using a similar process of converting a fraction into a percent. Express the ratio as a fraction, divide the numerator by the denominator and multiply the result by 100.

number of daisies : total number of flowers

$$8 : 18$$

$$\frac{8}{2} : \frac{18}{2}$$

$$4 : 9$$

Express the ratio as a percent.

$$4:9 = \frac{4}{9} = 4 \div 9 = 0.444...$$

Multiply the result by 100.

$$0.4444... \times 100 = 44.\overline{4}\%$$

You can make equivalent ratios by multiplying or dividing each of the terms in a given ratio by the same number.

Example 2

Identify 2 equivalent ratios for the given ratio 8 : 12.
To identify the first equivalent ratio, you can multiply both terms in the ratio by the same number. In this case, let's multiply both terms in the ratio by 2.

$8:12 \Rightarrow 8^{\times 2}:12^{\times 2} \Rightarrow 16:24$

To identify the second equivalent ratio, you can divide both terms in the ratio by a common factor. In this case, let's divide both terms in the ratio by 4.

$8:12 \Rightarrow 8^{\div 4}:12^{\div 4} \Rightarrow 2:3$

Two equivalent ratios for 8 : 12 are 16 : 24 and 2 : 3.

Practice Questions

1. There are 104 students in the school. If there are 56 girls in the school, what is the ratio of boys to girls? Express your answer in lowest terms.
   Step 1: Determine the number of boys in the school.
   104 – 56 = 48

   Step 2: Write the ratio of boys to girls.
   number of boys : number of girls

   $$48 : 56$$
   $$\frac{48}{8} : \frac{56}{8}$$
   $$6 : 7$$

   The ratio of boys to girls in the school is 6 : 7.

2. In a concrete mix, the ratio of sand to gravel is 3 : 5. To make 64 kg of concrete, how much sand is needed?

Step 1: Determine the part-to-whole ratio of sand to the total amount of ingredients in the concrete.

sand : total

3 : 8

Step 2: Determine the amount of sand needed.
Use the concept of equivalent ratios to determine the amount of sand needed.

$$\frac{3}{8} : \frac{?}{64}$$

When 8 is multiplied by 8, it results in 64. So, multiply 3 by 8 to determine the amount of sand needed.

$$\frac{3^{\times 8}}{8^{\times 8}} : \frac{24}{64}$$

3. Identify two equivalent ratios for the ratio 15 : 20.

$15 : 20 \Rightarrow 15^{\times 2} : 20^{\times 2} \Rightarrow 30 : 40$

$15 : 20 \Rightarrow 15^{\div 5} : 20^{\div 5} \Rightarrow 3 : 4$

Part 2: Problem Solving with Ratios

When problem solving with ratios, always set up a proportion first. Then apply cross products to solve for the unknown quantity

Example 1

An 18 m tree casts a 30 m shadow. How long is the shadow of a tree that is 27 m tall?

Step 1: Set up a proportion that represents the problem. Let x represent the unknown quantity.

$$\frac{\text{tree}}{\text{shadow}} \Rightarrow \frac{18}{30} = \frac{27}{x}$$

Step 2: Apply cross products to get the answer.

$$\frac{18}{30} = \frac{27}{x}$$
$$(18)(x) = (30)(27)$$
$$18x = 810$$
$$\frac{18x}{18} = \frac{810}{18}$$
$$x = 45$$

Example 2

A chainsaw requires fuel that is mixed in the ratio 0.2 L of oil to 1.6 L of gasoline. If 11.7 of fuel is needed, how much oil is required?

Step 1: Determine the part – to – whole ratio of oil to the total amount of ingredients in the fuel.

oil : fuel

0.2 : 1.8

Step 2: Determine the amount of oil needed. Apply cross products to get the answer.

$$\frac{0.2}{1.8} = \frac{x}{11.7}$$
$$(1.8)(x) = (0.2)(11.7)$$
$$1.8x = 2.34$$
$$\frac{1.8x}{1.8} = \frac{2.34}{1.8}$$
$$x = 1.3$$

The chainsaw requires 1.3L of oil to make the required amount of fuel.

Practice Questions

1. The ratio of red cars to black cars in a parking lot is 3 : 8. If there are 243 red cars, then how many black cars are there in the parking lot?

Step 1: Set up a proportion that represents the problem. Let x represent the unknown quantity.

$$\frac{red}{black} \Rightarrow \frac{3}{8} = \frac{243}{x}$$

Step 2: Apply cross products to get the answer.

$$\frac{3}{8} = \frac{243}{x}$$

$$(3)(x) = (8)(243)$$

$$3x = 1944$$

$$\frac{3x}{3} = \frac{1944}{3}$$

$$x = 648$$

There are 648 black cars in the parking lot.

2. In order to make fruit punch for her party, Marguerite mixes 2 cans of fruit juice with 5 cans of carbonated water. How much fruit punch will be made is she uses 10 cans of apple juice?

Step 1: Determine the part – to – whole ratio of fruit juice to the total amount of ingredients in the fruit punch.

fruit juice : fruit punch

2 : 7

Step 2: Determine the amount of punch needed.
Apply cross products to get the answer.

$$\frac{2}{7} = \frac{10}{x}$$

$$(2)(x) = (7)(10)$$

$$2x = 70$$

$$\frac{2x}{2} = \frac{70}{2}$$

$$x = 35$$

Marguerite will have made 35 cans of fruit punch.

Part 3: Working with Rates and Units Rates

A rate is a comparison of amounts of measurements that have different units. For example, a baker advertises $5.00 for 3 loaves of bread. In this case, money is being compared to the loaves of bread you can purchase. Rates are often expressed as unit rates, in which the second term is 1. It tells you how many units of the first quantity correspond to 1 unit of the second quantity. For example, Rosie was driving at 100 km/h or you pay $2.50/kg for a bag of apples.

Example 1

Nathan can jog 850 m in 3 min. At this rate how far can he jog in 1 hour?

Step 1: Set up a proportion to represent the situation. Let *x* represent the unknown quantity.

1 hour = 60 min

$$\frac{\text{m}}{\text{min}} \Rightarrow \frac{850}{3} = \frac{x}{60}$$

2 : 7

Step 2: Apply cross products to solve for the unknown quantity.

$$\frac{850}{3} = \frac{x}{60}$$

$$(3)(x) = (850)(60)$$

$$3x = 51\ 000$$

$$\frac{3x}{3} = \frac{51\ 000}{3}$$

$$x = 17\ 000$$

Nathan can jog 17 000 m in 1 hour.

Unit rates can be used to compare two situations to one another and determine which one is more efficient.

Example 2

Leslie travelled 630 km in 7 hours. Mark travelled 285 km in 3 hours. Who had the faster rate of speed?

Express each rate as a unit rate.

$$\textit{Leslie' speed}: \frac{630km}{7hours} = 90\ km/h$$

$$\textit{Mark' speed}: \frac{285km}{3hours} = 95\ km/h$$

Mark had the faster rate of speed.

Practice Questions

1. If 6 cupcakes cost $8.49, how much will 10 cupcakes cost?

Step 1: Set up a proportion to represent the situation. Let x represent the unknown quantity.

$$\frac{cupcakes}{cost} \Rightarrow \frac{6}{8.49} = \frac{10}{x}$$

Step 2: Apply cross products to solve for the unknown quantity.

$$\frac{6}{8.49} = \frac{10}{x}$$
$$(6)(x) = (8.49)(10)$$
$$6x = 84.90$$
$$\frac{6x}{6} = \frac{84.90}{6}$$
$$x = 14.15$$

It will cost $14.15 to purchase 10 cupcakes.

2. Which store has the best price for grapes? (Hint: Find the unit price per 100 g at each store).

Example

Store A	Store B	Store C
\$3.98 for 500 g	\$3.20 for 400 g	\$4.79 for 600 g

Store A	Store B	Store C
$\frac{\$3.98}{500g}=\frac{x}{100g}$ $(500)(x)=(3.98)(100)$ $500x = 398$ $\frac{500x}{500}=\frac{398}{500}$ $x = 0.796$	$\frac{\$3.20}{400g}=\frac{x}{100g}$ $(400)(x)=(3.20)(100)$ $400x = 320$ $\frac{400x}{400}=\frac{320}{400}$ $x = 0.80$	$\frac{\$4.79}{600g}=\frac{x}{100g}$ $(600)(x)=(4.79)(100)$ $600x = 479$ $\frac{600x}{600}=\frac{479}{600}$ $x = 0.798\overline{3}$

Store A has the best price for the grapes.

Lesson 2: Assignment

1. Identify two equivalent ratios for each of the following ratios.

   a. 24 : 36

   $\boldsymbol{24 : 36 \Rightarrow 24^{\div 12} : 36^{\div 12} \Rightarrow 2 : 3}$

   $\boldsymbol{24 : 36 \Rightarrow 24^{\div 3} : 36^{\div 3} \Rightarrow 8 : 12}$

   b. 14 to 35

   $\boldsymbol{14 \text{ to } 35 \Rightarrow 14^{\div 7} : 35^{\div 7} \Rightarrow 2 : 5}$

   $\boldsymbol{14 \text{ to } 35 \Rightarrow 14^{\times 2} : 35^{\times 2} \Rightarrow 28 : 70}$

   c. 16 :32 : 56

   $\boldsymbol{16 : 32 : 56 \Rightarrow 16^{\div 2} : 32^{\div 2} : 56^{\div 2} \Rightarrow 8 : 16 : 28}$

   $\boldsymbol{16 : 32 : 56 \Rightarrow 16^{\div 4} : 32^{\div 4} : 56^{\div 4} \Rightarrow 4 : 8 : 14}$

2. The ratio of the length of a rectangle to its width is 11 : 3. If the width is 12 cm, what is the length of the rectangle?

$$\frac{\text{length}}{\text{width}} \qquad \frac{11}{3} = \frac{n}{12}$$

$$(3)(n) = (11)(12)$$

$$3n = 132$$

$$\frac{3n}{3} = \frac{132}{3}$$

$$n = 44$$

The width of the rectangle is 44cm.

3. A building 21 m tall casts a shadow 16 m in length. How tall is a building that casts a shadow 63 m long? Round your answer to the nearest tenth of a metre.

$$\frac{actual}{shadow} \qquad \frac{21}{16} = \frac{n}{63}$$

$$(16)(n) = (21)(63)$$

$$16n = 1323$$

$$\frac{16n}{16} = \frac{1323}{16}$$

$$n = 82.6875$$

The building would be about 82.7 m tall.

4. In an election Marci received 8 votes for every 5 that Judy received. If 7384 people voted, how many votes did Marci get?

$$\frac{Marci}{total} \qquad \frac{8}{13} = \frac{m}{7384}$$

$$(13)(m) = (8)(7384)$$

$$13m = 59\ 072$$

$$\frac{13m}{13} = \frac{59\ 072}{13}$$

$$m = 4544$$

Marci received 4544 votes.

5. Bill and Ted are two business partners who share their profits in a ratio of 6 : 5. If they make a profit of $6 880.50, what is each partners share?

Step 1: Determine the amount of money Bill will get.
Set up a proportion that represents the situation and solve for the unknown.

$$\frac{Bill}{total} \qquad \frac{6}{11} = \frac{b}{6880.50}$$
$$(11)(b) = (6)(6880.50)$$
$$11b = 41\ 283$$
$$\frac{11b}{11} = \frac{41\ 283}{11}$$
$$b = \$3753.00$$

Step 2: Determine the amount of money Ted will get.
Subtract the amount of money Bill gets from the total profit.
$6880.50 - $3753 = $3127.50
Bill will get $3753.00 and Ted will get $3127.50.

6. Grapefruits are on sale at 3 for $2.19. At this rate, how much would 8 grapefruits cost?

$$\frac{cost}{grapefruit} \qquad \frac{\$2.19}{3} = \frac{g}{8}$$
$$(3)(g) = (2.19)(8)$$
$$3g = 17.52$$
$$\frac{3g}{3} = \frac{17.52}{3}$$
$$g = \$5.84$$

The cost of 8 grapefruits is $5.84.

7. Linda scored 63 points in 4 basketball games. At this rate, approximately how many points would she score in 11 games? Round your answer to the nearest whole number.

$$\frac{\textit{points}}{\textit{game}} \qquad \frac{63}{4} = \frac{x}{11}$$

$$(4)(x) = (63)(11)$$

$$4x = 693$$

$$\frac{4x}{4} = \frac{693}{4}$$

$$x = 173.25$$

Linda can score approximately 173 points in 11 games.

8. Which of the following cereals is the best buy?

Example

Store A	Store B	Store C
\$4.89 for 1.2 kg	\$7.38 for 1.8 kg	\$2.47 for 600 g

Store A	Store B	Store C
$\frac{\$4.89}{1200g} = \frac{x}{100g}$ $(1200)(x) = (4.89)(100)$ $1200x = 489$ $\frac{1200x}{1200} = \frac{489}{1200}$ $x = \$0.4075$	$\frac{\$7.38}{1800g} = \frac{x}{100g}$ $(1800)(x) = (7.38)(100)$ $1800x = 738$ $\frac{1800x}{1800} = \frac{738}{1800}$ $x = \$0.41$	$\frac{\$2.47}{600g} = \frac{x}{100g}$ $(600)(x) = (2.47)(100)$ $600x = 247$ $\frac{600x}{600} = \frac{247}{600}$ $x = \$0.411\overline{6}$

Store A has the best price for cereal.

9. Margo earned \$53.50 for 5 hours of work. How much will she earn if she works 36 hours?

$$\frac{\textit{money earned}}{\textit{hour}} \qquad \frac{\$53.50}{5} = \frac{x}{36}$$

$$(5)(x) = (53.50)(36)$$

$$5x = 1926$$

$$\frac{5x}{5} = \frac{1926}{5}$$

$$x = \$385.20$$

Margo will earn \$385.20 if she works 36 hours.

10. A car used 15 litres of gasoline to travel 300 km. At this rate, how many litres of gasoline are needed to travel a distance of 1056 km?

$$\frac{\textit{litres}}{\textit{distance}} \qquad \frac{15}{300} = \frac{x}{1056}$$

$$(300)(x) = (15)(1056)$$

$$300x = 15\ 840$$

$$\frac{300x}{300} = \frac{15\ 840}{300}$$

$$x = 52.8$$

It will require 52.8L of gas to travel 1056 km.