Important Concepts . . .

Preview Review

Mathematics Grade 8 TEACHER KEY

W2 - Lesson 2: Solving Linear Equations

Important Concepts of Grade 8 Mathematics

W1 - Lesson 1 ..... Perfect Squares and Square Roots
W1 - Lesson 2 ..... Working with Ratios and Rates
W1 - Lesson 3 ..... Multiplying and Dividing Fractions
W1 - Lesson 4 ..... Multiplying and Dividing Integers
W1 - Lesson 5 ..... Working with Percents
W1 - Review
W1 - Quiz

W2 - Lesson 1 ..... Modelling and Solving Linear Equations Using Algebra Tiles
W2 - Lesson 2 ..... Solving Linear Equations
W2 - Lesson 3 ..... Graphing and Analyzing Linear Relations
W2 - Lesson 4 ..... Critiquing the Representation of Data
W2 - Lesson 5 ..... Probability of Independent Events
W2 - Review
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W3 - Lesson 1 ..... Pythagorean Theorem
W3 - Lesson 2 ..... Calculating Surface Area
W3 - Lesson 3 ..... Calculating Volume
W3 - Lesson 4 ..... Drawing 3-D Objects
W3 - Lesson 5 ..... Congruence of Polygons
W3 - Review
W3 - Quiz

Materials Required

Protractor
Ruler
Calculator

No Textbook Required

This is a stand-alone course.

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Preview/Review Concepts for Grade Eight Mathematics

Teacher Key

W2 – Lesson 2:

Solving Linear Equations

OBJECTIVES

By the end of this lesson, you will be able to:

- Solve a one-step and two-step linear equation
- Verify the solution to a given linear equation
- Apply the distributive property to solve a given linear equation
- Solve a given problem using a one-step or two-step linear equation

GLOSSARY

Coefficient – the number located directly in front of the variable

Constant – a whole number that is either added to or subtracted from the variable

Distributive Property – a mathematical property that states that a product can be written as a sum or difference of two products, $a\ (b + c) = ab + ac$ or $a\ (b - c) = ab - ac$

Inverse Operations – an operation that reverses the effect of another operation. The inverse of addition is subtraction, the inverse of subtraction is addition, the inverse of multiplication is division, and the inverse of division is multiplication.

Linear Equation – equations that form a straight non-vertical and non-horizontal line when graphed

One-step Equations – equations that can be solved in one-step

Two-step Equations – equations that involve two steps in order to solve them

Variable – a symbol (usually a letter) that represents an unknown value or values

W2 – Lesson 2: Solving Linear Equations

Materials required:

- Paper, and Pencil

Part 1: Solving One-Step Linear Equations

When solving one-step equations, the goal is to isolate the variable.
In order to do this, you must apply the inverse operation to both sides of the equation.
Verify your solution by substituting the result back into the equation and evaluating.
Ensure that the left side of the equation equals the right side of the equation.

Example 1

Solve for n in the following equation 3n = -12.
Apply the inverse operation to both sides of the equation to remove the coefficient.

$$3n = -12$$

The inverse of multiplication is division.

$$\frac{\cancel{3}n}{\cancel{3}} = \frac{-12}{3}$$

$$n = -4$$

Divide both sides of the equation by 3.

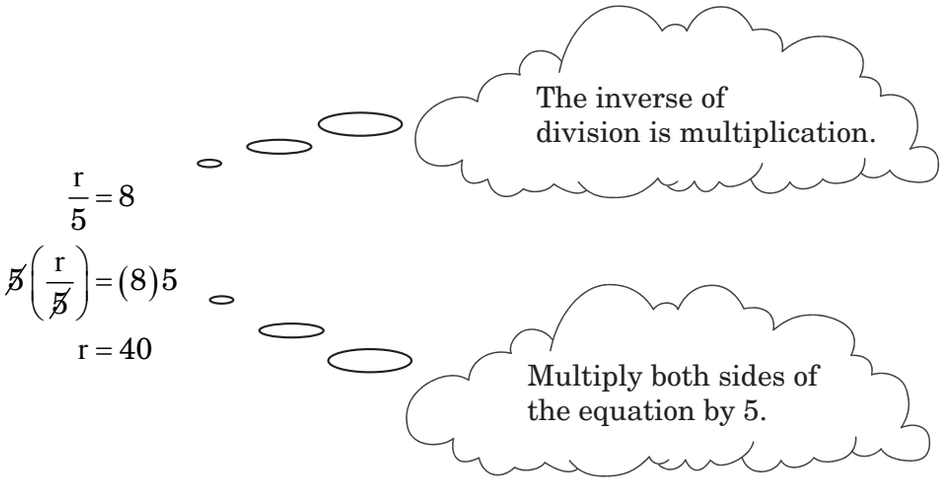
Verify the solution.

$$3n = -12$$
$$3(-4) = -12$$
$$-12 = -12$$
$$LS = RS$$

Example 2

Solve for r in the following equation $\frac{r}{5} = 8$.

Apply the inverse operation to both sides of the equation to isolate the variable.



Verify the solution.

$$\frac{r}{5} = 8$$
$$\frac{40}{5} = 8$$
$$8 = 8$$
$$LS = RS$$

Practice Questions

1. $-13p = 39$

$$-13p = 39$$
$$\frac{-13p}{-13} = \frac{39}{-13}$$
$$p = -3$$

2. $\frac{b}{8} = 4$

$$\frac{b}{8} = 4$$
$$8\left(\frac{b}{8}\right) = (4)8$$
$$b = 32$$

3. $-10y = -120$

$$-10y = -120$$
$$\frac{-10y}{-10} = \frac{-120}{-10}$$
$$y = 12$$

4. $\frac{s}{-10} = 15$

$$\frac{s}{-10} = 15$$
$$(-10)\left(\frac{s}{-10}\right) = (15)(-10)$$
$$s = -150$$

Part 2: Solving Two-Step Linear Equations

Solving two-step equations involves a process similar to that of solving one-step linear equations. The main goal is to isolate the variable.

The first inverse operation will involve adding or subtracting the constant from the term that contains the variable.

The second inverse operation will involve multiplication or division to remove the numerical coefficient from the variable.

Remember to apply the inverse operation to both sides of the equation.

Verify your solution by substituting the result back into the equation and evaluating. Ensure that the left side of the equation equals the right side of the equation.

Example 1

Solve for n in the following equation 5n - 1 = 9.
Apply the first inverse operation to remove the constant.

$$5n - 1 = 9$$
$$5n - 1 + 1 = 9 + 1$$
$$5n = 10$$

The inverse of subtraction is addition.

Add 1 to both sides of the equation.

Apply the second inverse operation to remove the numerical coefficient.

$$5n = 10$$
$$\frac{5n}{5} = \frac{10}{5}$$
$$n = 2$$

The inverse of multiplication is division.

Divide both sides of the equation by 5.

verify the solution.

$$5n - 1 = 9$$
$$5(2) - 1 = 9$$
$$10 - 1 = 9$$
$$9 = 9$$
$$LS = RS$$

Example 2

Solve for b in the following equation $\frac{b}{-2}+9=13$.

Apply the first inverse operation to remove the constant.

$$\frac{b}{-2}+9=13$$

The inverse of addition is subtraction.

$$\frac{b}{-2}+9-9=13-9$$

$$\frac{b}{-2}=4$$

Subtract 9 from both sides of the equation.

Apply the second inverse operation to remove the numerical coefficient.

$$\frac{b}{-2}=4$$

The inverse of division is multiplication.

$$(-2)\left(\frac{b}{-2}\right)=(4)(-2)$$

$$b=-8$$

Multiply both sides of the equation by -2.

Verify the solution.

$$\frac{b}{-2}+9=13$$

$$\frac{-8}{-2}+9=13$$

$$4+9=13$$

$$13=13$$

$$LS=RS$$

Practice Questions

1. $\frac{r}{4} - 8 = 2$

$$\frac{r}{4} - 8 = 2$$
$$\frac{r}{4} - 8 + 8 = 2 + 8$$
$$\frac{r}{4} = 10$$
$$4\left(\frac{r}{4}\right) = (10)4$$
$$r = 40$$

2. $-2b + 4 = -20$

$$-2b + 4 = -20$$
$$-2b + 4 - 4 = -20 - 4$$
$$-2b = -24$$
$$\frac{-2b}{-2} = \frac{-24}{-2}$$
$$b = 12$$

3. $\frac{z}{4} + 10 = -13$

$$\frac{z}{4} + 10 = -13$$
$$\frac{z}{4} + 10 - 10 = -13 - 10$$
$$\frac{z}{4} = -23$$
$$4\left(\frac{z}{4}\right) = (-23)4$$
$$z = -92$$

4. $12c - 8 = -44$

$$\begin{aligned} 12c - 8 &= -44 \\ 12c - 8 + 8 &= -44 + 8 \\ 12c &= -36 \\ \frac{12c}{12} &= \frac{-36}{12} \\ c &= -3 \end{aligned}$$

Part 3: Applying the Distributive Property

The distributive property states that a product can be written as a sum or difference of two products, $a(b + c) = ab + ac$ or $a(b - c) = ab - ac$. Multiply each term inside the brackets by the term located outside the brackets.

Example 1

Simplify the algebraic expression 3 (x + 10).
Multiply each term inside the brackets by the 3.

$$\begin{aligned} 3(x+10) &= 3(x)+3(10) \\ &= 3x+30 \end{aligned}$$

Sometimes a linear equation will need to be simplified before it can be solved. Apply the distributive property before applying the inverse operations when solving two-step linear equations.

Example 2

Solve the linear equation 4 (x - 8) = -24.
Apply the distributive property to remove the brackets.

$$\begin{aligned} 4(x-8) &= -24 \\ 4(x)-4(-8) &= -24 \\ 4x+32 &= -24 \end{aligned}$$

Multiply each term in the brackets by 4.

Apply inverse operations to remove the constant.

$$\begin{aligned} 4x+32-32 &= -24-32 \\ 4x &= -56 \end{aligned}$$

Subtract both sides of the equation by 32.

Apply inverse operations to remove the numerical coefficient.

$$\begin{aligned} \frac{4x}{4} &= \frac{-56}{4} \\ x &= -14 \end{aligned}$$

Divide both sides of the equation by 4.

Practice Questions

1. $7(x+12)=56$

$$
\begin{aligned}
7(x+12)&=56\\
7(x)+7(12)&=56\\
7x+84&=56\\
7x+84-84&=56-84\\
7x&=-28\\
\frac{7x}{7}&=\frac{-28}{7}\\
x&=-4
\end{aligned}
$$

2. $-3(x-10)=48$

$$
\begin{aligned}
-3(x-10)&=48\\
-3(x)-3(-10)&=48\\
-3x+30&=48\\
-3x+30-30&=48-30\\
\frac{-3x}{-3}&=\frac{18}{-3}\\
x&=-6
\end{aligned}
$$

Part 4: Problem Solving with Linear Equations

When solving problems translate the situation represented in the problem into a linear equation. Then solve for the unknown.

Example 1

Laura bought 5 hockey sticks for $65.00. Each stick was on sale for $8.00 off. How many sticks did Laura purchase in total?

Step 1: Translate the situation represented in the problem into a linear equation.
Let x represent the original price of the hockey sticks.
Each hockey stick was on sale for $8.00 off and Laura bought 5 of them. This can be expressed as 5(x -8).

Laura spent $65.00 in total. The equation becomes 5(x - 8) = 65.

Step 2: Solve for the unknown in the linear equation.
Apply the distributive property.

$$\begin{aligned} 5(x-8) &= 65 \\ 5(x)-5(8) &= 65 \\ 5x-40 &= 65 \end{aligned}$$

Apply inverse operations to isolate the variable.

$$\begin{aligned} 5x-40 &= 65 \\ 5x-40+40 &= 65+40 \\ 5x &= 105 \\ \frac{5x}{5} &= \frac{105}{5} \\ x &= 21 \end{aligned}$$

The hockey sticks were originally priced at $21.00.

Example 2

Rena is 6 years older than twice Aaron's age. How old is Aaron if Rena is 22 years old?

Step 1: Translate the situation represented in the problem into a linear equation.
Let a represent Aaron's age.
Rena is 6 years older than twice Aaron's age can be expressed as 2a + 6.
Rena is 22 years old. The equation becomes 2a + 6 = 22.

Step 2: Solve for the unknown in the linear equation.
Apply inverse operations to isolate the variable.

$$\begin{aligned} 2a + 6 &= 22 \\ 2a + 6 - 6 &= 22 - 6 \\ 2a &= 16 \\ \frac{2a}{2} &= \frac{16}{2} \\ a &= 8 \end{aligned}$$

Aaron is 8 years old.

Practice Questions

1. On a fishing trip, Bethany caught two more fish than three times the number of fish Alex caught. If Bethany caught eight fish, how many did Alex catch?

   Step 1: Translate the situation represented in the problem into a linear equation.
   Let f represent the number of fish Alex caught.
   Bethany caught two more fish than three times the number of fish Alex caught can be expressed as 3 f + 2.
   Bethany caught eight fish. The equation becomes 3f + 2 = 8.

   Step 2: Solve for the unknown in the linear equation.
   Apply inverse operations to isolate the variable.

$$\begin{aligned} 3\mathrm{f} + 2 &= 8 \\ 3\mathrm{f} + 2 - 2 &= 8 - 2 \\ 3\mathrm{f} &= 6 \\ \frac{3\mathrm{f}}{3} &= \frac{6}{3} \\ \mathrm{f} &= 2 \end{aligned}$$

   Alex caught 2 fish.

2. Heather is lifting weights in the exercise room. She puts on 6 equal sized weights on her bar. The total amount Heather is lifting is 84 kg. The clamps that hold each weight in place have a mass of 1 kg, how much does each weight weigh?

Step 1: Translate the situation represented in the problem into a linear equation.
Let w represent the mass of each weight.
Heather puts on 6 equal sized weights on her bar and each clamps has a mass of 1 kg can be expressed as 6(w + 1).
Heather is lifting is 84 kg. The linear equation becomes 6(w + 1) = 84.

Step 2: Solve for the unknown in the linear equation.
Apply the distributive property.

$$\begin{aligned} 6(w+1) &= 84 \\ 6(w)+6(1) &= 84 \\ 6w+6 &= 84 \end{aligned}$$

Apply inverse operations to isolate the variable.

$$\begin{aligned} 6w+6 &= 84 \\ 6w+6-6 &= 84-6 \\ 6w &= 78 \\ \frac{6w}{6} &= \frac{78}{6} \\ w &= 13 \end{aligned}$$

Each weight has a mass of 13 kg.

Lesson 7: Assignment

Solve the following linear equations.

1. $2a = 20$

$$\frac{2a}{2} = \frac{20}{2}$$
$$a = 10$$

2. $\frac{z}{5} = -16$

$$5\frac{z}{5} = (-16)5$$
$$z = -80$$

3. $-15x = 45$

$$\frac{-15x}{-15} = \frac{45}{-15}$$
$$x = -3$$

4. $-9t = -27$

$$\frac{-9t}{-9} = \frac{-27}{-9}$$
$$t = 3$$

5. $\frac{a}{2} + 1 = -4$

$$\frac{a}{2} + 1 - 1 = -4 - 1$$
$$\frac{a}{2} = -5$$
$$2\left(\frac{a}{2}\right) = (-5)2$$
$$a = -10$$

6. $2x - 7 = 9$

$$2x - 7 + 7 = 9 + 7$$
$$2x = 16$$
$$\frac{2x}{2} = \frac{16}{2}$$
$$x = 8$$

7. $18 = 6 - 3a$

$$18 - 6 = 6 - 6 - 3a$$
$$12 = -3a$$
$$\frac{12}{-3} = \frac{-3a}{-3}$$
$$-4 = a$$

8. $\frac{b}{12} - 4 = 1$

$$\frac{b}{12} - 4 + 4 = 1 + 4$$
$$\frac{b}{12} = 5$$
$$12\left(\frac{b}{12}\right) = (5)12$$
$$b = 60$$

9. $10 + 7r = 31$

$$10 - 10 + 7r = 31 - 10$$
$$7r = 21$$
$$\frac{7r}{7} = \frac{21}{7}$$
$$r = 3$$

10. $-6+\frac{a}{3}=5$

$$-6+6+\frac{a}{3}=5+6$$
$$\frac{a}{3}=11$$
$$3\left(\frac{a}{3}\right)=(11)3$$
$$a=33$$

11. $3(x-7)=-27$

$$3(x)-3(7)=-27$$
$$3x-21=-27$$
$$3x-21+21=-27+21$$
$$3x=-6$$
$$\frac{3x}{3}=\frac{-6}{3}$$
$$x=-2$$

12. $-5(v+9)=-60$

$$-5(v)-5(9)=-60$$
$$-5v-45=-60$$
$$-5v-45+45=-60+45$$
$$-5v=-15$$
$$\frac{-5v}{-5}=\frac{-15}{-5}$$
$$v=3$$

13. $-4(x-2)=36$

$$\begin{aligned}
-4(x)-4(-2)&=36\\
-4x+8&=36\\
-4x+8-8&=36-8\\
-4x&=28\\
\frac{-4x}{-4}&=\frac{28}{-4}\\
x&=-7
\end{aligned}$$

14. $-8(m+7)=48$

$$\begin{aligned}
-8(m)-8(7)&=48\\
-8m-56&=48\\
-8m-56+56&=48+56\\
-8m&=104\\
\frac{-8m}{-8}&=\frac{104}{-8}\\
m&=-13
\end{aligned}$$

15. Dale is three times as old as Salima. If Dale is 36, how old is Salima?

Let s represent Salima's age.
If Dale is three times as old as Salima and is 36 years old, the linear equation that represents this problem is $3s=36$.

$$\begin{aligned}
3s&=36\\
\frac{3s}{3}&=\frac{36}{3}\\
s&=12
\end{aligned}$$

Salima is 12 years old.

16. Maple has $5.00 less than one third of the money that Chan has. If Maple has $25.00, how much money does Chan have?

Let c represent the amount of money Chan has.
If Maple has $5.00 less than one third of the money that Chan has and Maple has $25.00, the linear equation that represents this problem is $\frac{c}{3} - 5 = 25$.

$$\frac{c}{3} - 5 + 5 = 25 + 5$$
$$\frac{c}{3} = 30$$
$$3\left(\frac{c}{3}\right) = (30)3$$
$$c = 90$$

Chan has $90.00.

17. Vancouver averages 9 less than 4 times the number of rainy days (with 5 mm or more) than Edmonton does. If Vancouver gets 123 days of rain, how many rainy does Edmonton get?

Let e represent the number of rainy days Edmonton gets.
If Vancouver averages 9 less than 4 times the number of rainy days than Edmonton and Vancouver gets 123 days of rain, the linear equation that represents this problem is $4e - 9 = 123$.

$$4e - 9 = 123$$
$$4e - 9 + 9 = 123 + 9$$
$$4e = 132$$
$$\frac{4e}{4} = \frac{132}{4}$$
$$e = 33$$

Edmonton gets 33 days of rain.

18. Jagar had three baseball cards. He bought six more packages. He now has 51 cards. How many cards were in each package if each package contained the same number?

Let p represent the number of baseball cards in each package.
If Jagar had three baseball cards and buys six more packages and now has 51 cards, the linear equation that represents this problem is $6\mathrm{p}+3=51$.

$$6\mathrm{p}+3-3=51-3$$
$$6\mathrm{p}=48$$
$$\frac{6\mathrm{p}}{6}=\frac{48}{6}$$
$$\mathrm{p}=8$$

There are 8 cards in each package.

19. In Canada, we measure temperature in degrees Celsius (°C). In the United States, temperature is measured in degrees Fahrenheit (°F). The temperature measured in Fahrenheit is 32 more than 1.8 times the temperature measured in Celsius. If the temperature in Seattle is 77°F, what would it be expressed as in degrees Celsius?

Let C represent the temperature in degrees Celsius.
If temperature measured in Fahrenheit is 32 more than 1.8 times the temperature measured in Celsius and the temperature in degrees Fahrenheit is 77, the linear equation that represents this problem is $77=1.8\mathrm{C}+32$.

$$77=1.8\mathrm{C}+32$$
$$77-32=1.8\mathrm{C}+32-32$$
$$45=1.8\mathrm{C}$$
$$\frac{45}{1.8}=\frac{1.8\mathrm{C}}{1.8}$$
$$25=\mathrm{C}$$

The same temperature in Canada would be expressed as 25°C.

20. On Saturday, Damaris went to the zoo with three of her friends. The zoo reduces its admission prices on Saturdays by $4.00. In total, the girls spent $32.00. What would be the cost of admission had the girls gone to the zoo on Friday?

Let x represent the cost of admission at the zoo on Friday.
If Damaris went to the zoo with three of her friends, the admission prices on Saturdays are reduced by $4.00, and in total, the girls spent $32.00, the linear equation that represents this problem is $4(x-4)=32$.

$$\begin{aligned} 4(x)-4(4) &= 32 \\ 4x-16 &= 32 \\ 4x-16+16 &= 32+16 \\ 4x &= 48 \\ \frac{4x}{4} &= \frac{48}{4} \\ x &= 12 \end{aligned}$$

The original price of admission at the zoo is $12.00.


LEARNING
ANYTIME ANYWHERE